

NOISE POWER

$$\tilde{n} = \int_{-\infty}^{\infty} n(T-\tau) h(\tau) d\tau$$

Noise

Noise after passing through receive filter and sampling at $t = T$.

$$E\{|\tilde{n}|^2\} = E\{n^2\}$$

assuming real elements or quantities.

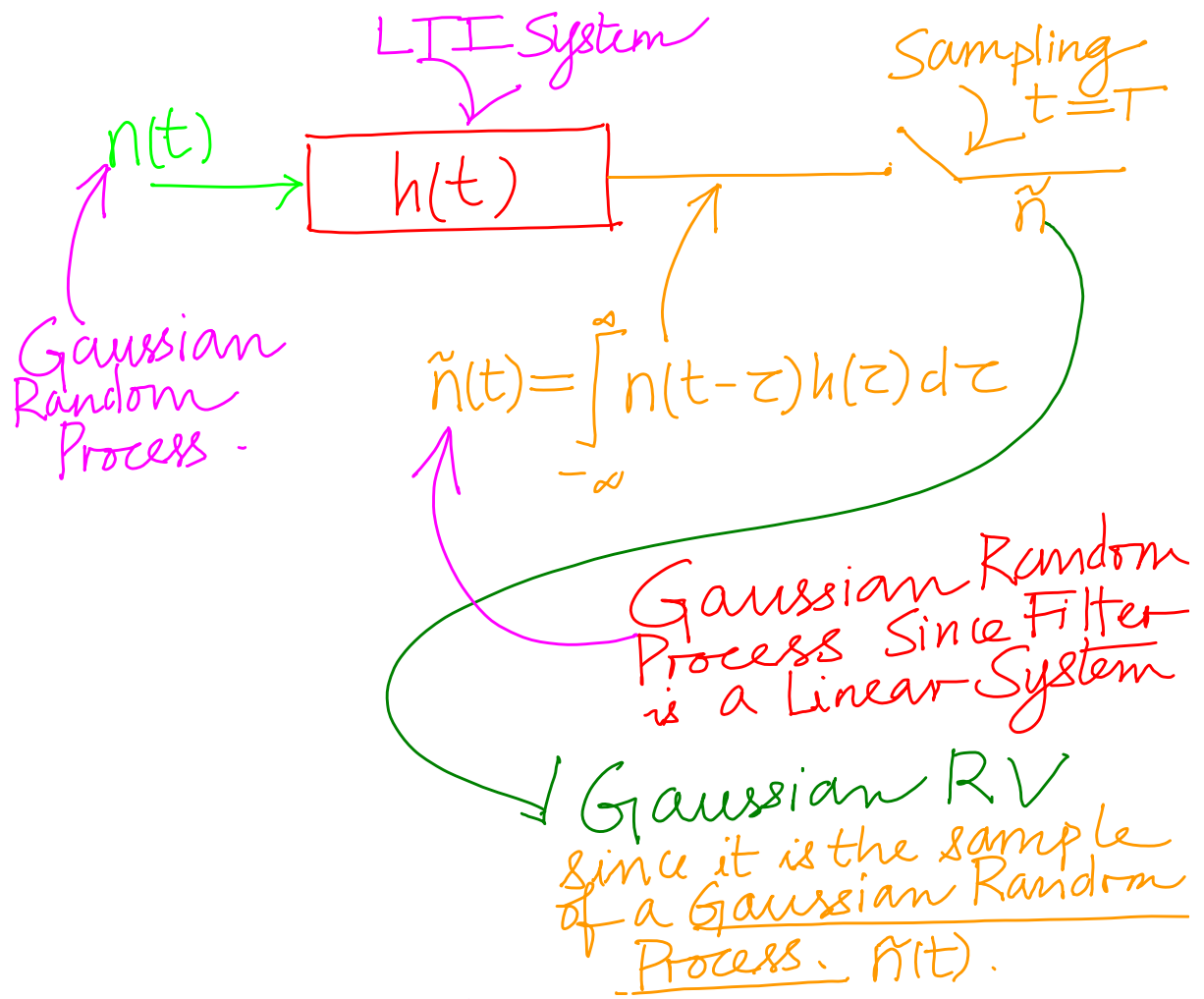
$n(t)$ ← White + Gaussian

AutoCorrelation

$$E\{n(t)\} = 0$$

$$E\{n(t)n(t+\tau)\} = R_{nn}(\tau) = \frac{\eta_0}{2} \delta(\tau)$$

Zero-mean



The sample $\tilde{n}$ is given by the integral:

$$\tilde{n} = \int_{-\infty}^{\infty} n(T-\tau) h(\tau) d\tau$$

$\tilde{n}$ is Gaussian.

Characterized by mean, variance,

$$E\{\tilde{n}\} = ?$$

mean?

$$E\{\tilde{n}\} = E\left\{\int_{-\infty}^{\infty} n(T-\tau)h(\tau)d\tau\right\}$$

Expectation Integral
are Linear Operations

$$\Rightarrow = \int_{-\infty}^{\infty} \underbrace{E\{n(T-\tau)\}}_0 h(\tau)d\tau$$

Deterministic
Fixed.

0 since $n(t)$ is
Zero-mean
process.

$$E\{\tilde{n}\} = 0$$

Zero-mean

$$E\{\tilde{n}^2\} = ?$$

Variance
or Noise Power.

$$E\{\tilde{n}^2\} = E\{\tilde{n} \cdot \tilde{n}\}$$

$$= E\left\{ \int_{-\infty}^{\infty} n(T-\tau) h(\tau) d\tau \times \int_{-\infty}^{\infty} n(T-\tilde{\tau}) \cdot h(\tilde{\tau}) d\tilde{\tau} \right\}$$

Simply change variable of integration

$$= E\left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} n(T-\tau) n(T-\tilde{\tau}) \cdot \frac{h(\tau) h(\tilde{\tau})}{d\tau \cdot d\tilde{\tau}} \right\}$$

Putting integrals Together

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{E\{n(T-\tau) \cdot n(T-\tilde{\tau})\}}{d\tau \cdot d\tilde{\tau}} \cdot h(\tau) \cdot h(\tilde{\tau})$$

auto correlation
 $E\{n(t)n(t+\tau)\}$

$$= \frac{\eta_0}{2} \delta(\tau)$$

Depends on Time Difference τ .

$$\frac{\eta_0}{2} \delta(\tau - \tilde{\tau})$$

$$= \frac{\eta_0}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(z) h(\tilde{z}) \delta(\tilde{z} - z) d\tilde{z} dz.$$

inner integral wrto $\tilde{z}$
outer integral wrto z

$$= \frac{\eta_0}{2} \int_{-\infty}^{\infty} h(z) \left(\int_{-\infty}^{\infty} h(\tilde{z}) \delta(\tilde{z} - z) d\tilde{z} \right) dz$$

$h(z)$.

$$\int_{-\infty}^{\infty} x(t) \delta(t - t_0) dt = x(t_0).$$

Sifting Property
of Dirac Delta

$$= \frac{\eta_0}{2} \int_{-\infty}^{\infty} |h(z)|^2 dz$$

Energy of Filter E_h .

$$E\{\tilde{n}^2\} = \frac{\eta_0}{2} E_h.$$

Noise Power

variance of $\tilde{n}$

At output
of Sampler

$$E\{\tilde{n}^2\} = \frac{\eta_0}{2} \int_{-\infty}^{\infty} |h(z)|^2 dz$$